Simple Harmonic Motion

Periodic Motion: A motion which repeats itself over and over again after a regular interval of time is called Periodic motion.

" Any motion which repeats itself after regular interval is called Periodic or harmonic motion.

Ex. Motion of hands of clock, Motion of earth around the sun, Motion of a Piston in a cylinder, Motion of a ball in a bowl, Motion of a liquid in a U-tube.

Oscillatory ~~and~~ or Vibratory Motion:- When a body moves to and fro on either side of a point in definite time interval, then this motion is said to be oscillatory or vibratory motion.

Ex. Motion of mass suspended from a spring, motion of simple Pendulum.

* each Vibratory motion is Periodic but each Periodic motion is not vibratory.

Time Period & frequency:- The time after which the body retraces its path is called the time Period and the number of vibrations made in one second is called frequency.

Time Period $\boxed{T = \frac{2\pi}{\omega}}$ frequency $\boxed{f = \frac{1}{T} = \frac{\omega}{2\pi}}$

Displacement and Amplitude:- The Physical quantity which varies uniformly with time in an oscillatory motion is called displacement. The maximum value of displacement is known as amplitude.

Simple Harmonic Motion:- A motion in which the acceleration of the body is proportional to its displacement from the mean position and is always directed towards the mean position is known as the simple harmonic motion. It has following characteristics:

(i) Motion is on both sides of mean position. The maximum displacement on one side of mean position is known as amplitude.

(b) The body repeats its motion in a definite interval of time.

(c) Acceleration is always proportional to the displacement and is directed opposite to it.

(d)* In case of S.H.M. time Period is independent of amplitude.

* The necessary and sufficient condition for a motion to be simple harmonic is that restoring force or torque must be linear. i.e.

$F = -ky$ or $\tau = -C\theta$

(Linear S.H.M.) (Angular S.H.M.)

Differential Equation for S.H.M.

$$\boxed{\frac{d^2y}{dt^2} + \omega^2 y = 0}$$

ω - angular frequency.
y - displacement at time 't'

Displacement Equation for S.H.M.

$$\boxed{y = a\sin\omega t \quad \text{or} \quad y = a\sin(\omega t + \phi)}$$

where ωt or $(\omega t + \phi)$ is called the Phase of the Particle at any instant 't' and ϕ is the initial Phase of the Particle.

* If two particles executing S.H.M. cross their mean positions in same direction, then they are in same Phase of Vibration. If they cross the mean position in opposite directions then they are in opposite phase. The phase difference between two particles in same phase is 2π while the phase difference between the two particles in opposite phase is π.

Velocity of Particle executing S.H.M.

$$v = \frac{dy}{dt} = \frac{d}{dt}(a\sin\omega t)$$

$$\boxed{v = a\omega\cos\omega t} \quad \text{or} \quad \boxed{v = \omega\sqrt{a^2 - y^2}}$$

At mean position $y = 0$	$\therefore v_{max} = a\omega$
At Extreme position $y = a$,	$\therefore v_{min} = 0$

Acceleration of Particle executing S.H.M.

$$\alpha = \frac{dv}{dt} = \frac{d}{dt}(a\omega\cos\omega t)$$

$$\boxed{\alpha = -a\omega^2\sin\omega t = -\omega^2 y}$$

At mean position $y = 0$	$\therefore \alpha_{min} = 0$
At Extreme position $y = a$	$\therefore \alpha_{max} = -\omega^2 a$

Time Period: $\alpha = \omega^2 y$

or $\omega^2 = \frac{\alpha}{y}$

or $\omega = \sqrt{\alpha/y}$

or $\frac{2\pi}{T} = \sqrt{\alpha/y}$

or $T = 2\pi\sqrt{y/\alpha}$

or $$\boxed{T = 2\pi\sqrt{\text{displacement}/\text{acceleration}}}$$

Phase: — Phase is that Property of wave motion which tells us the position of the particle at any instant. Phase is measured either by the angle which particle makes with the mean position or by fraction of time Period or by fraction of wavelength.

* When a body is in S.H.M. then the Phase difference between velocity and acceleration and velocity and displacement is 90°.
* The Phase difference between acceleration and displacement is 180°.

1. A particle executes a S.H.M. with an amplitude a. when the Particle is at its extreme Position, its Phase is

(a) $\pi/5$ (b) $\pi/4$ (c) $\pi/3$ (d) $\pi/2$.

Solution:- $y = a \sin(\omega t + \phi)$

for extreme Position $t = 0$, $y = a$

then $a = a \sin \phi$

or $\sin \phi = 1$ or $\phi = \pi/2$.

2. If in the equation $y = 5 \sin(0.05t + 0.05)$, y represents the displacement at the instant t, then which one of the following gives the frequency?

(a) 5 (b) 0.05 (c) $0.05/2\pi$ (d) $2\pi \times 0.05$

Solution:- $y = a \sin(\omega t + \phi)$

$y = 5 \sin(0.05t + 0.05)$

$\omega = 0.05$ frequency $f = \frac{\omega}{2\pi} = \frac{0.05}{2\pi}$

3. The displacement of a particle is represented by

$y = 10 \sin(5t + \theta)$. The Period of oscillation is:—

(a) $2\pi/5$ sec (b) $5/2\pi$ sec (c) $\pi/5$ sec. (d) $5/\pi$ sec.

Solution:- $y = a \sin(\omega t + \phi)$

$y = 10 \sin(5t + \theta)$

$\omega = 5$ $T = \frac{2\pi}{\omega} = 2\pi/5$ sec.

4. A Particle is acted simultaneously by two mutually perpendicular S.H. Ms;

$x = a \cos\omega t$ and $y = a \sin\omega t$. The trajectory of motion of the Particle will be:

(a) an ellipse (b) a Parabola

(c) a circle (d) a hyperbola.

Solution: $x = a \cos\omega t$, $y = a \sin\omega t$

$x^2 + y^2 = a^2$ (It is a Circle)

5. A body of mass 5 gm is executing S.H.M. about a fixed Point O. With an amplitude of 10 cm, its maximum velocity is 100 cm/sec. Its velocity will be 50 cm/sec at a distance

(a) 5 cm (b) $5\sqrt{2}$ cm (c) $5\sqrt{3}$ cm (d) $10\sqrt{2}$ cm.

Solution:- $v_{max} = a\omega = 100$

$\omega = 100/a = \frac{100}{10}$ [$\because a = 10$

$= 10 rd/sec.$

$v^2 = \omega^2(a^2 - y^2)$

$(50)^2 = (10)^2[(10)^2 - y^2]$

$25 = 100 - y^2$ or $y^2 = 75$

or $y = 5\sqrt{3}$ cm.

6. A simple pendulum performs S.H.M. about $x = 0$ with an amplitude a, and time period T. The speed of pendulum at $x = a/2$ will be:

(a) $\pi \cdot \frac{a\sqrt{3}}{T}$ (b) $\frac{\pi a\sqrt{3}}{2T}$ (c) $\frac{\pi a}{T}$ (d) $\frac{3\pi^2 a}{T}$

Solution:- $\omega = \frac{2\pi}{T}$, $x = \frac{a}{2}$.

$v = \omega\sqrt{a^2 - x^2} = \frac{2\pi}{T} \cdot \sqrt{a^2 - \frac{a^2}{4}}$

$v = \frac{2\pi}{T} \cdot \frac{\sqrt{3}a}{2} = \frac{\pi a\sqrt{3}}{T}$

7. The maximum acceleration of a body moving in S.H.M is a_0 and maximum velocity is v_0. The amplitude is given by:

(a) v_0^2/a_0 (b) $a_0 v_0$ (c) a_0^2/v_0 (d) $1/a_0 v_0$

Solution: $v_{max} = A\omega = v_0$

$A^2\omega^2 = v_0^2$

$\alpha_{max} = A\omega^2 = a_0$

$\frac{A^2\omega^2}{A\omega^2} = v_0^2/a_0$

or $A = v_0^2/a_0$

8. In the above problem, the time period of oscillation will be given by:

(a) $2\pi v_0/a_0$ (b) $2\pi a_0/v_0$ (c) $2\pi a_0 v_0$ (d) $2\pi/a_0 v_0$

Solution:- $A\omega = v_0$

$A\omega^2 = a_0$

$\frac{A\omega^2}{A\omega} = \frac{a_0}{v_0}$

$\omega = \frac{a_0}{v_0}$

$\frac{2\pi}{T} = \frac{a_0}{v_0}$

$\frac{T}{2\pi} = \frac{v_0}{a_0}$

$T = \frac{2\pi v_0}{a_0}$

9. For a particle in S.H.M. if the amplitude of displacement is a and the amplitude of velocity is v, the amplitude of acceleration is

(a) va (b) v^2/a (c) $v^2/2a$ (d) v/a.

Solution:- amplitude of displacement $= a$.

amplitude of velocity $v_{max} = a\omega = v$ — (1)

amplitude of acceleration $\alpha_{max} = a\omega^2 = (a\omega)\cdot\omega$ — (2)

from (1) $a^2\omega^2 = v^2$ — (3)

such that $\frac{a^2\omega^2}{a} = \frac{v^2}{a}$ or $a\omega^2 = v^2/a$ — (3)

from (2) & (3) $\alpha_{max} = v^2/a$.

10. A particle executes S.H.M. along a straight line so that its time period is 12 sec. The time it takes in ~~transversing~~ traversing a distance equal to half its amplitude from its equilibrium is:

(a) 6 sec (b) 4 sec (c) 2 sec (d) 1 sec.

Solution:- $y = a\sin\omega t$

$\because y = a/2$

$\therefore \frac{a}{2} = a\sin\frac{2\pi}{T}\cdot t$

$\frac{1}{2} = \sin\frac{2\pi}{12}\cdot t$

$\sin\pi/6 = \sin\frac{\pi t}{6}$

or $\frac{\pi t}{6} = \pi/6$ or $t = 1$ sec

11. The time period of a particle undergoing S.H.M. is 16 sec. It starts motion from the mean position. After 2 sec, its velocity is 0.4 m/sec. The amplitude is

(a) 1.44 m (b) 0.72 m (c) 2.88 m (d) 0.36 m.

Solution: $v = a\omega\cos\omega t = a\cdot\frac{2\pi}{T}\cdot\cos\frac{2\pi}{T}\cdot t$

$0.4 = \frac{2\pi a}{16}\cdot\cos\frac{2\pi}{16}\cdot 2$

$0.4 = \frac{\pi a}{8}\cdot\cos\pi/4$

$\frac{4}{10} = \frac{\pi a}{8}\times\frac{1}{\sqrt{2}}$

$a = \frac{32\sqrt{2}}{10\pi} = 1.44$ m

12. A Particle undergoes simple harmonic motion having time Period T. The time taken in $\frac{3}{8}$th oscillation is:

(a) $\frac{3}{8}T$ (b) $\frac{5}{8}T$ (c) $\frac{5}{12}T$ (d) $\frac{7}{12}T$

Solution:- Time to complete $\frac{1}{4}$th oscillation is T/4 sec. Time to complete $\frac{1}{8}$th vibration from extreme Position is obtained from

$$y = \frac{\cancel{a}}{2} = a\cos\omega t = \cancel{a}\cos\frac{2\pi}{T}.t$$

$$\text{or } \cos\frac{\pi}{3} = \cos\frac{2\pi}{T}.t$$

$$\text{or } \frac{\pi}{3} = \frac{2\pi}{T}.t$$

$$\text{or } t = \frac{T}{6}$$

So time to complete $\frac{3}{8}$th oscillation

$$t' = \frac{T}{4} + \frac{T}{6} = \frac{5T}{12}\text{ sec}$$

13. The maximum speed of a particle executing S.H.M. is 1m/sec and maximum acceleration is 1.57 m/sec^2. Its time Period is

(a) 4 sec (b) 2 sec (c) 1.57 sec (d) 1/1.57 sec.

Solution:- $v_{max} = a\omega = \cancel{1.5}$ — ①

$\alpha_{max} = a\omega^2 = 1.57$ — ②

$$\cancel{\frac{a\omega}{a\omega^2}} = \frac{1}{1.57}$$

$$\frac{1}{\omega} = \frac{1}{1.57}$$

$$\frac{T}{2\pi} = \frac{1}{1.57} \text{ or } T = \frac{2\pi}{1.57} = \frac{\cancel{2}\times \overset{2}{\cancel{3.14}}}{\cancel{1.57}} = 4\text{ sec}$$

14. The maximum speed of a particle executing S.H.M. is 1m and maximum acceleration is 1.57 m/sec^2. Its frequency is

(a) 0.25 sec^{-1} (b) 2 sec^{-1} (c) 1.57 sec^{-1} (d) 2.57 sec^{-1}.

Solution:- $v_{max} = a\omega = 1$; $\alpha_{max} = a\omega^2 = 1.57$

$$\omega = \frac{a\omega^2}{a\omega} = \frac{1.57}{1} = 1.57$$

$$\text{frequency } f = \frac{\omega}{2\pi} = \frac{1.57}{2\pi} = \frac{1}{4} = 0.25\text{ sec}^{-1}.$$

15. Two Particles are executing S.H.M of same amplitude and frequency along the same straight line path. They pass each other when going in ~~opposition~~ opposite directions, each time their displacement is half of their amplitude. What is the phase difference between them?

(a) $5\pi/6$ (b) $2\pi/3$ (c) $\pi/3$ (d) $\pi/6$.

Solution:- $y = a\sin(\omega t + \phi_1)$; when $y = a/2$, then

$\frac{a}{2} = a\sin(\omega t + \phi_1)$ or $\sin(\omega t + \phi_1) = \sin \pi/6$

or $\omega t + \phi_1 = \pi/6$.

if $y = -a/2$ then

$-\frac{a}{2} = a\sin(\omega t + \phi_2)$ or $\sin(\omega t + \phi_2) = \sin 5\pi/6$

or $\omega t + \phi_2 = 5\pi/6$

Phase diff $\phi_2 - \phi_1 = (\omega t + \phi_2) - (\omega t - \phi_1)$

$= \frac{5\pi}{6} - \frac{\pi}{6} = \frac{4\pi}{6} = 2\pi/3$

16. The time taken by a particle executing S.H.M. of Period 'T' to move from the mean position to half the maximum displacement is:

(a) $T/2$ (b) $T/4$ (c) $T/8$ (d) $T/12$.

Solution:- $y = a\sin\omega t$

$\frac{a}{2} = a\sin\frac{2\pi}{T}t$ or $\sin\frac{2\pi}{T}t = \sin\pi/6$

or $\frac{2\pi}{T}t = \frac{\pi}{6}$

or $t = T/12$

17. A particle moves such that its acceleration 'a' is given by $a = -bx$ where x is the displacement from equilibrium position and b is constant. The Period of oscillation is:

(a) $2\pi/b$ (b) $2\pi/\sqrt{b}$ (c) $\sqrt{2\pi/b}$ (d) $2\sqrt{\pi/b}$.

Solution:- $T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}} = 2\pi\sqrt{\frac{\cancel{x}}{b\cancel{x}}}$

$T = 2\pi/\sqrt{b}$

18. Two simple harmonic motions are represented by the following equations :

$y_1 = 10 \sin\left[\frac{\pi}{4}(12t+1)\right]$; $y_2 = 5\left[\sin 3\pi t + \sqrt{3}\cos 3\pi t\right]$

find the ratio of their amplitudes.

(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 2 : $\sqrt{3}$

Solution :- $y_1 = 10 \sin\left[\frac{\pi}{4}(12t+1)\right]$

$a_1 = 10$

$y_2 = 5\sin 3\pi t + 5\sqrt{3}\cos 3\pi t$

$a_2 \cos\phi = 5$, $a_2 \sin\phi = 5\sqrt{3}$

$a_2^2 = 25 + 75 = 100$

$a_2 = 100$

$a_1 : a_2 = 1 : 1$

19. The acceleration d^2x/dt^2 of a particle varies with displacement x as $\frac{d^2x}{dt^2} = -Kx$, where K is a constant of the motion. The time period 'T' of the motion is equal to

(a) $2\pi K$ (b) $2\pi\sqrt{K}$ (c) $2\pi/\sqrt{K}$ (d) $2\pi/K$.

Solution :- differential equation of S.H.M. is

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \quad \text{or} \quad \frac{d^2x}{dt^2} = -\omega^2 x \quad \text{—} (1)$$

given equation $$\frac{d^2x}{dt^2} = -Kx \quad \text{—} (2)$$

Compare (1) & (2) $\omega^2 = K$ or $\omega = \sqrt{K}$

or $\frac{2\pi}{T} = \sqrt{K}$

or $T = \frac{2\pi}{\sqrt{K}}$.

20. The motion of a particle varies with time according to the relation $y = a(\sin\omega t + \cos\omega t)$, it is represented —

(a) Motion is oscillatory but not S.H.M.

(b) the motion is S.H.M. with amplitude $a\sqrt{2}$

(c) the motion is S.H.M. with amplitude a

(d) the motion is S.H.M. with amplitude 2a.

21. Which one of the following equations does not represent S.H.M. x = displacement and t = time. Parameters a, b and c are constants of motion

(a) $x = a\sin bt$ (b) $x = a\cos bt + C$

(c) $x = a\sin bt + c\cos bt$ (d) $x = a\sec bt + c\,\mathrm{cosec}\,bt$

22. Which of the following is not simple Harmonic function?

(a) $y = a\sin 2\omega t + b\cos 2\omega t$ (b) $y = a\sin\omega t + b\cos 2\omega t$

(c) $y = 1 - 2\sin^2\omega t$ (d) $y = \sqrt{a^2+b^2}\cdot\sin\omega t\cos\omega t$

23. The motion of a particle executing S.H.M. is given by $x = 0.01\sin 100\pi(t+0.05)$ where x is in meters and time t is in seconds. The time period is

(a) 0.2 sec (b) 0.1 sec (c) 0.02 sec (d) 0.01 sec.

Solution :– $x = a\sin(\omega t+\phi)$

$x = 0.01\sin 100\pi(t+0.05)$

$\omega = 100\pi$ or $\frac{2\cancel{\pi}}{T} = 100\cancel{\pi}$

or $T = \frac{2}{100} = 0.02\,sec$

24. The displacement y of a particle executing periodic motion is given by $y = 4\cos^2 t/2 \cdot \sin 1000t$. This expression may be considered to be a result of the superposition of how many independent harmonic motions?

(a) Five (b) Two (c) Three (d) four.

Solution :– $y = 4\cos^2 t/2 \cdot \sin 1000t$

$y = 2\cdot[1+\cos t]\cdot\sin 1000t$

$y = 2\sin 1000t + 2\cos t\cdot\sin 1000t$

$y = 2\sin 1000t + \sin 1001t + \sin 999t$

It is superposition of three independent harmonic motions.

25. A particle of mass 1 kg is moving in S.H.M. with an amplitude 0.02 m and a frequency of 60 Hz. The maximum force in newton acting on the particle is :–

(a) $188\pi^2$ (b) $144\pi^2$ (c) $288\pi^2$ (d) None of the above.

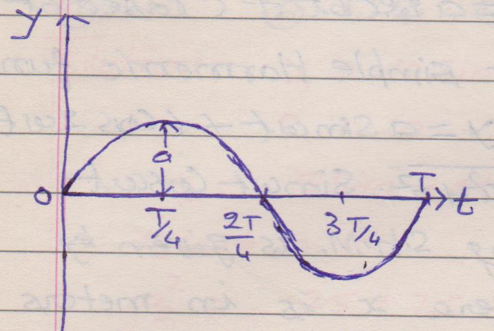
Solution :– $F_{max} = m\cdot\alpha_{max} = m\cdot\omega^2 a = m\,4\pi^2\nu^2 a$

$= 1\times 4\pi^2\times 0.02\times(60)^2 = \cancel{0.0288\pi^2} = 288\pi^2$

—x—

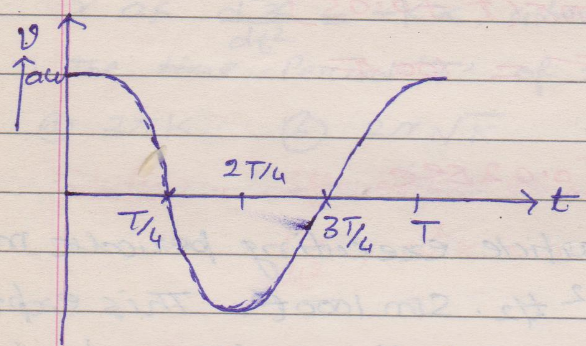
Graphical Variation of displacement, velocity, accelera- and time in S.H.M.

(i) displacement – time graph:–



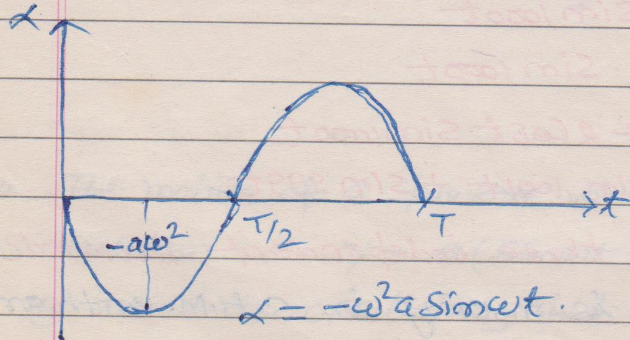
$y = a \sin \omega t$

(ii) Velocity – time graph:–



$v = a\omega \cos \omega t$

(iii) Acceleration – time – graph:–



$\alpha = -\omega^2 a \sin \omega t$